PHILOSOPHY AND NON-EUCLIDEAN GEOMETRY.

By F. A. Foraker.

Leibnitz and Descartes made remarkable contributions to both mathematics and philosophy. Newton obtains a high rank in the history of the former subject, but only a minor place in the history of philosophy, while Kant, who possessed a well-founded knowledge of the science and mathematics of his time, receives one of the foremost positions in the history of philosophy. Upon the basis of these facts, if we neglect a few of the lesser lights, the statement is often made that there is a relationship between the study of mathematics and the study of philosophy, and that he who studies one of them will also find himself a devotee in the pursuit of the other.

This is certainly not true in general of modern mathematicians and philosophers. The field covered by each of these groups has become so broad that an intimate knowledge of the other is not so easily obtained as in the days when Leibnitz wrote his calculus and his monadology.

The philosopher searches for "a comprehensive veiw of nature, an attempt at a universal explanation of things" in the words of Weber,* but in his explanations of things mathematical, at least, he has often shown that he is not familiar with the thing to be explained. Non-Euclidean geometry has crumpled many philosophical theories, but the philosopher is seldom aware of it, because he has very little knowledge, if any, of this branch of mathematics. Some of them write as if they were in entire ignorance of the existence of such a subject, others have mentioned it or commented upon it in such a manner as to show that they comprehended very little of its meaning.

Another example of this fact has recently been set forth in a work entitled "Psychology of High School Subjects."† An extract follows:

* "History of Philosophy."

† Charles Hubbard Judd.

"There is no difficulty whatsoever in talking about parallel lines that meet. In the world of complete abstraction, that is, in the world of thought about lines, we may think of lines in any way that seems to us to be desirable; but we cannot take the results of such thought back into the world of practice or even into the world of direct imagery. Anyone who, after speculation for a time about different kinds of possible space, ***should try to lay railroad rails, on the assumption that parallel lines meet, would find himself in difficulty.*** Speculation and practical life thus frequently need to be distinguished from each other. Indeed, the distinction is so evident that perhaps the better statement of the case is to say that the two forms of experience need reconciliation."

The mathematician may very well agree with the statement that abstract thinking leads us into regions where we should endeavor to find reconciliation with the world of reality, or he might even go further, and say that there is no assurance that we shall always find realities corresponding to the results of abstract thinking, but the author mentioned has chosen to illustrate his meaning by a very unfortunate example.

The implication of the passage from which this extract is taken seems to be that railroad rails can only be laid on the Euclidean hypothesis of parallel lines. If we are going to seek for correspondences between the results of our abstract thinking and the realities of the world, we must be sure that we get as near as possible to what those realities are, or we may write into our philosophy some errors which will be as serious as any we might retain if we did not attempt to secure such a correspondence.

The fact concerning the laying of railroad rails is that it is impossible to lay them by the Euclidean hypothesis. The author quoted talks as if he believed that in laying rails, a separate line is surveyed for each rail. Now, that is not very close to reality. Even if the earth were a plane and if the Euclidean hypothesis were demonstratively true, it would still remain impossible because instruments and men are not without error. If we were to grant the possibility of perfect instruments and the ability of men to handle them without error, still Euclidean geometry could not lay the rails, for the earth is spheroidal, and any two

straight lines that Euclidean geometry could lay on such a surface must intersect twice. If the line for one of the rails is straight and the line of the second is everywhere equidistant from it, then the second line is not a straight one *but non-Euclidean geometry can lay an equidistant line as well as Euclidean geometry.*

When we lay the rails of a railroad, a measuring bar of constant length is continually slid along between the rails as they are put down, and the problem is not one in Euclidean geometry, but the manner of actually placing the rails shows that this process is independent of any hypothesis concerning parallel lines. Moreover, it is evident that a mathematician would not use the phrase taken from this quotation, "in talking about *parallel lines that meet*" when he speaks of non-Euclidean geometry. The mathematician might say that parallel lines are *lines that do not meet* or that parallel lines are lines that meet in infinity, but even the latter definition could not enter into the method of laying rails for a railroad on a finite earth.

This recalls a quotation from William James's chapter on Necessary Truths. "If we mean by a parallel a line that will never meet a second line; and if we have one such line drawn through a point, any new line drawn through that point which does not coalesce with the first must be inclined to it, *and if inclined to it, must approach the second, i. e.,* cease to be parallel with it. No properties of outlying space need come in here; only a definite conception of uniform direction, and constancy in sticking to one's point."

Let us neglect the last clause which probably never had any definite meaning, even in the mind of James. It is evident from the glaring fallacy given here in italics that one who discusses parallelism between mental and cerebral phenomena is not necessarily competent to discuss the parallelism of lines. This is, perhaps, one of Kant's original intuitions *a priori* to all study of geometry of the last century. Do philosophers study the mathematics of the present time, or are they content to quote Aristotle and St. Augustine on this branch of human knowledge? Do mathematicians do better with philosophy?

UNIVERSITY OF PITTSBURGH,
PITTSBURGH, PA.